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Half a Century of Bell's Theorem

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Chapter 28
Bell's Theorem, Bell Inequalities, and the "Probability Normalization Loophole"

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Abstract Fifty years ago in 1964, John Bell [6], showed that deterministic local hidden-variables theories are incompatible with quantum mechanics for idealized systems. Inspired by his paper, Clauser, Horne, Shimony and Holt (CHSH) [12] in 1969 provided the first experimentally testable Bell Inequality and proposed an experiment to test it. That experiment was first performed in 1972 by Freedman and Clauser [20]. In 1974 Clauser and Horne (CH) [13] first showed that all physical theories consistent with "Local Realism" are constrained by an experimentally testable loophole-free Bell Inequality—the CH inequality. These theories were further clarified in 1976–1977 in "An Exchange on Local Beables", a series of papers by Bell, Shimony, Horne, and Clauser [8] and by Clauser and Shimony (CS) [15] in their 1978 review article. In 2013, nearly fifty years after Bell's original 1964 paper [6], two groups, Giustina et al. [24] and Christensen et al. [11] have finally tested the loophole-free CH inequality. Clauser and Shimony (CS) [15] also showed that the CHSH inequality is testable in a loophole-free manner by using a "heralded" source. It was first tested this way by Rowe et al. [35] in 2001, and more convincingly in 2008 by Matsukevich et al. [33]. To violate a Bell Inequality and thereby to disprove Local Realism, one must experimentally examine a two component entangled-state system, in a configuration that is analogous to a *Gedankenexperiment* first proposed by Bohm [9] in 1951. To be used, the configuration must generate a normalized coincidence rate with a large amplitude sinusoidal dependence upon adjustable apparatus settings. Proper normalization of this amplitude is critical for the avoidance of counterexamples and loopholes that can possibly invalidate the test. The earliest tests used the CHSH inequality without source heralding. The first method for normalizing coincidence rates without heralding was proposed by CHSH [12] in 1969. It consists of an experimental protocol in which coincidence rates measured with polarizers removed are used to

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normalize coincidence rates measured with polarizers inserted. Very high transmission polarizers are required when using this method. Highly reasonable and very weak supplementary assumptions by CHSH and by CH allow this protocol to work in a nearly loophole free manner. A second method for normalizing coincidence rates was offered by Garuccio and Rapisarda [22] in 1981. As will be discussed below, it allows experiments to be done more easily, but at a significant cost to the generality of their results. It was first used in the experiment by Aspect, Grangier, and Roge [3] in 1982. It uses "ternary-result" apparatuses and allows the use of highly absorbing polarizers, which would not work with other normalization methods. It normalizes using a sum of coincidence rates. Gerhardt et al. [23] in 2011 theoretically and experimentally demonstrated counterexamples for tests that use this normalization method. Their experiments thus obviate the validity of their counterexamples, and further indicate that very high transmission polarizers are necessary for convincing tests to be performed.

Introduction to Bell's Theorem and the Bell Inequalities

Bell's Theorem is formulated in terms of a set of individually named inequalities, each with increasing generality and scope. These inequalities are collectively referred to as the "Bell Inequalities". They surprisingly follow from very simple natural assumptions concerning the nature of reality. These assumptions, along with their associated consequences via Bell's Theorem, then constitute a minimal framework for a whole class of theories originally named "Objective Local Theories" by Clauser and Horne (CH) [13] in 1974, and subsequently renamed "Local Realism" by Clauser and Shimony (CS) [15] in 1978. The assumptions underlying Local Realism are so simple and natural that one of this conference's organizers, Anton Zeilinger, recently commented that if Bell's Theorem had been discovered before quantum mechanics, it would have been promoted to be considered a law of nature on its own, whereupon the subsequently discovered quantum mechanics must obviously be wrong! The assumptions underlying Local Realism are reviewed in the Appendix.

The essence of Bell's Theorem is that theories based on Local Realism cannot give the same prediction for certain "entangled-state" two-component systems as does the theory of quantum mechanics. Thus, these two opposing theories are experimentally distinguishable from each other. It is then the task for experimental physicists to determine which of these two incompatible theories correctly describes the world in which we live. To refute Local Realism (and/or to refute quantum mechanics) experimentally, one can perform an experiment whose quantum mechanical predictions violate a Bell Inequality. The first experiment to do so was that by Freedman and Clauser [20] in 1972. Their results were then the first to violate the CHSH inequality (but still leave open the normalization and locality loopholes). Freedman and Clauser's experimental results have been overwhelmingly confirmed by many other experiments, some of which are discussed and tabulated in the section "Some Experimental Results". The experiment by Aspect, Dailbard and Roger [4] again violated the CHSH inequality, and was first to close the locality loophole, but still leave open the normalization loophole. The normalization-loophole-free heralded-source CHSH inequality was first tested by Rowe et al. [35] in 2001, and more convincingly in 2008 by Matsukevich et al. [33], with both experiments still leaving open the locality loophole. Giustina et al. [24] and Christensen et al. [11] have finally tested the normalization-loophole-free CH inequality in 2013. Experiments are currently in progress to finally close both the normalization loophole and the locality loophole simultaneously in a single experiment.

The particular entangled-state two-component systems referred to above were used in a *Gedankenexperiment* that was first envisaged by David Bohm [9] in 1951. That entangled-state system was used by John Bell [6] in his now-famous 1964 paper. Bohm's *Gedankenexperiment* is described below in the section "Bohm's 1951 *Gedankenexperiment* and Its Relation to Bell's Theorem". Bohm's arrangement then provides a prototype configuration for Bell's Theorem experiments.

The generality and scope of Bell's Theorem has evolved since its discovery. John Bell's 1964 paper [6] introduced the first Bell-Inequality and showed that no Local Hidden Variables Theory (LHVT) can give the quantum mechanical prediction for Bohm's idealized *Gedankenexperiment*. In that paper, however, Bell made no reference to the experimental status, or even to the experimental testability of his result. Moreover, Bell's 1964 [6] Inequality applied only to idealized systems. To bridge the gap between theory and the real world in which we live, Clauser, Horne, Shimony, and Holt (CHSH) [12] in 1969 introduced the second Bell Inequality—the CHSH inequality—and showed that it holds for deterministic LHVT's that govern realizable systems. More importantly, unlike Bell's 1964 result, the CHSH inequality is experimentally testable. CHSH also were the first to propose an actual experiment to test the Theorem's predictions.

The extension of Bell's Theorem to include Objective Local Theories (and Local Realism) was made by Clauser and Horne (CH) [13] in 1974, and these theories were renamed Local Realism by Clauser and Shimony [15] in 1978. Clauser and Horne therein introduced what is now commonly referred to as the CH inequality. It is experimentally testable and is loophole free, and is what we herein refer to as an "R-inequality" (see below). A further discussion clarifying the meaning and scope of Bell's Theorem followed CH in "An Exchange on Local Beables" [8]—a series of papers by Bell, Shimony, Horne and Clauser. A review of the various proofs and interpretations of Bell's Theorem, the various Bell Inequalities, and the available modalities for experimental testing is given by the Clauser and Shimony (CS) [15] review article.

To violate a Bell inequality and thereby to disprove Local Realism, one must experimentally examine a two component entangled-state system, in a configuration that is analogous to Bohm's *Gedankenexperiment*. The experiment is done with two widely separated apparatuses. To be used for a test, the configuration must generate normalized coincidence rates at these two apparatuses with a measured large amplitude sinusoidal dependence upon the two adjustable apparatus settings. Proper normalization of the coincidence rates is critical for the avoidance a loophole.

A loophole exists when a counterexample exists that invalidates the experimental test. Loopholes sometimes arise when technology limits just how closely one can approach the ideal experiment specified by a Bell Inequality. Various experimental tricks are then generally used, along with associated supplementary assumptions to plug these loopholes. These added assumptions generally do not rely on either locality or realism (or quantum mechanics), although it is highly desirable that they at least be consistent with these theories. To evaluate the assumptions, one may examine how reasonable a supplementary assumption is, along with how contrived the associated counterexample is. Such assumptions thus become the weak point in any argument claiming an experimental disproof of Local Realism. The obvious question is always offered—are you testing the fundamental assumptions behind Local Realism, or are you just testing the supplementary assumption(s)? Fortunately, recent experiments closing the remaining loopholes are now rendering this last question moot.

The first identified loophole is the so-called "locality loophole". Curiously, it was first noted by Bohm and Aharonov [10] in 1957, prior to Bell's 1964 paper [6]. Under the locality loophole, a hypothetical collusion between the two separated apparatuses can possibly occur, whereby the apparatuses communicate their settings to each other. Such communication can then possibly can account for the strange quantum-mechanical predictions associated with the entanglement of widely separated particles. This possibility was promoted further by Bell in his 1964 paper [6]. Bohm and Aharonov had suggested that a rapid change of the two apparatuses of Bohm's *Gedankenexperiment*, while the entangled-state particles are in flight, can thereby exclude any such collusion. While this locality-loophole counterexample may seem somewhat contrived, it has become particularly important to close it when Bell's Inequality experimental results are under attack by malevolent efforts, as may occur when the experimental outcomes are used for quantum communication and cryptography, for which malevolent forces are well known to exist (e.g. by eavesdroppers). The first experiment to close this loophole was performed in 1982 by Aspect, Dalibard, and Roger [4].

The second identified loophole is what we herein call the "normalization loophole". It occurs when the measured large amplitude sinusoidal dependence on adjustable apparatus settings is less than that required for an actual violation of a Bell Inequality. The so-called "detection loophole" is one of several examples of the normalization loophole. The detection (normalization) loophole commonly occurs when low detection efficiency reduces the measured amplitude of the coincidence-rate variation to below that needed for a violation of a Bell Inequality.

Sometimes the normalization loophole occurs without it even being recognized. Indeed, the transition from Bell's 1964 inequality [6] to the CHSH inequality [12] involved closure of the first example of a normalization loophole, wherein Bell assumed and indeed required a perfect apparatus correlation. (See the sections "One Possible Cause for the Normalization Loophole" and "Bell's 1964 E-Inequality for Idealized Binary Result Apparatuses" below.) In general, the normalization loophole can be closed only with highly precise apparatus, and with careful count-rate normalization. It has only been closed recently. Closure of both the locality loophole and the normalization loophole simultaneously in a single experiment has not yet been done, but experiments now in progress promise to do so soon.

Much of the remainder of this paper addresses the normalization loophole. There are two known routes to plug it—either violate the CHSH inequality using a heralded source, as was first suggested by Clauser and Shimony (CS) [15] in 1978, or directly violate the CH inequality [13]. Both routes have met significant technical difficulties, and both routes require highly efficient detection schemes and highly efficient high-transmission analyzers (polarizers).

In the section "Bohm's 1951 *Gedankenexperiment* and Its Relation to Bell's Theorem", we describe Bohm's 1951 *Gedankenexperiment* [9]. It provides a basic prototype for Bell's Theorem experiments. In the section "One Possible Cause for the Normalization Loophole", we discuss the origin of the normalization loophole. The loophole's nature depends on the nature of the associated Bell Inequality being tested. In deriving a Bell Inequality, there are at least two ways to proceed. One way is to start directly from observed quantities, such as the number of observed particle detections (per unit time), then to calculate probabilities for them, and finally to derive an inequality constraining them that is consistent with the requirements for locality and realism. This path was followed by CH. It yields the CH Inequality, and the experiments that it constrains are then inherently free from the normalization-loophole. It is described in the section "Normalization-Loophole Free Clauser-Horne (CH) R-Inequality for Binary-Result Apparatuses". The CH inequality uses binary-result apparatuses. The 1978 review article by Clauser and Shimony (CS) [15] describes a variety of alternative methods for deriving the CH inequality. The CH inequality directly constrains observed count rates, and is thus, what we herein call an R-inequality, as an abbreviation for count-Rate-inequality. An R-inequality directly compares one linear combination of measured count rates with another.

A second method for deriving a Bell Inequality is that originally followed by Bell and by CHSH. It requires one to first define "result values" (as discussed in the section "Result Values and Expectation-Value Inequalities (E-Inequalities)"). That method then provides inequalities that constrain the expectation values for the various observed results. We call these "E-inequalities",[1] as an abbreviation for Expectation-value-inequalities. The first such E-inequality was derived by Bell in his original 1964 paper [6]. It is discussed in the section "Bell's 1964 E-Inequality for Idealized Binary Result Apparatuses". The second E-inequality was that by CHSH [12]. It is discussed in the section "Clauser Horne Shimony Holt (CHSH) E-Inequality for Real Binary Result Apparatuses". Unfortunately, an E-inequality is not directly testable, unless it is first converted to an R-inequality. Care must be exercised when one is performing the conversion in order to avoid unnecessarily introducing a normalization loophole.

[1]Historical Note: Both Bell [6] and CHSH [12] use the symbol P rather than E for the expectation value of the product of the binary result values A and B.Subsequent works generally now use the symbol E.

Care must also be exercised in recognizing whether or not one is using binary-result or ternary-result apparatuses. (See the section "One Possible Cause for the Normalization Loophole") Three methods have been used for doing the conversion. One method, described in section "Clauser Horne Shimony Holt (CHSH) R-Inequalities for Real Binary-Result Apparatuses via the CHSH Polarizer-Removal Protocol", is to use the CHSH polarizer-removal protocol to get a CHSH R-inequality. It requires the use of binary-result apparatuses, and it was used for all of the earliest tests of the CHSH E-and R-inequalities. The protocol consists of measuring coincidence rates with polarizers removed as well as measuring coincidence rates with polarizers inserted. The former measurements are used to normalize the latter. Very high transmission polarizers are required for this method to work. Highly reasonable supplementary assumptions by CHSH and by CH allow this protocol to provide a very reasonable argument for avoidance of the loophole. Clauser and Horne (CH) [13], however, do provide an *ad hoc* somewhat contrived counterexample, discussed in the section "The CH Counterexample". Thus, a residual normalization loophole remains with this method, despite the high plausibility of their associated supplementary assumption and the rather contrived nature of that counterexample.

A second method for converting an E-inequality into an R-inequality is described in the section "CHSH R-Inequality with Heralding". It uses a heralded source, and it was first explicitly suggested by Clauser and Shimony (CS) [15] in 1978. It then allows one to use the CHSH E-inequality directly to get a loophole-free CHSH R–inequality. It can use either binary- or ternary-result apparatuses.

A third method for normalizing coincidence rates was first proposed in 1981 by Garuccio and Rapisarda (GR) [22], and was first used in 1982 in the experiment by Aspect, Grangier, Roger (AGR) [3]. (See the section "Garuccio and Rapisarda/Aspect Grangier Roger R-Inequalities for Real Ternary-Result Apparatuses".) It uses "ternary-result" apparatuses only. Notably, it allows the use of highly absorbing polarizers, whereby a violation of an associated R-inequality is much easier to achieve experimentally. It normalizes the coincidence rates using a sum of these coincidence rates, and ignores unobserved (and unobservable) events. Unlike the CHSH polarizer-removal protocol, no polarizers are removed using this method, and no additional normalizing data need be taken. It also requires a much stronger supplementary assumption than that required by the CHSH polarizer-removal protocol. The GR/AGR supplementary assumption is now commonly (and gratuitously) referred to as the "fair-sampling assumption". Gerhardt et al. [23] provide a convincing experimental demonstration of the ease by which it can be countered, especially by malevolent efforts, as may occur in "security related scenarios" and quantum cryptography. Despite the need for these strong supplementary assumptions, GR/AGR normalization has been used by many experiments, presumably because of it's ease of experimental implementation. Its use has become sufficiently common that it is often cited (incorrectly) as an integral necessary part of the CHSH E-inequality, despite strident protestations made by this author at the first Quantum [Un]speakables conference (Clauser [16]). It is not!

We conclude in the section "Some Experimental Results" with a description and tabulation of various experimental results that test the predictions made by these various Bell Inequalities.

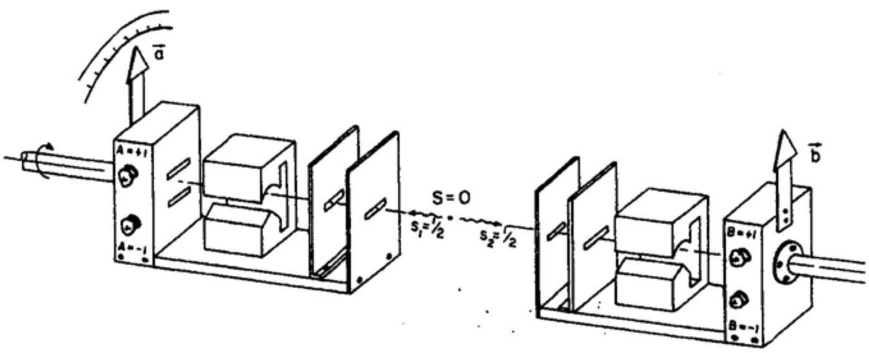
Bohm's 1951 *Gedankenexperiment* and Its Relation to Bell's Theorem

Figure 28.1 shows Bohms's 1951 *Gedankenexperiment* [9]. It provides the prototype for Bell's-Theorem discussions. It consists of a highly idealized pair of binary-result apparatuses interacting with a quantum mechanically entangled two-particle system. In this *Gedankenexperiment*, a spin-zero particle decays into a pair of spin-entangled spin-½ particles. Each of these particles, in turn, flies into an associated rotatable Stern-Gerlach analyzer, where it then follows one of two trajectories, and is detected by one of two associated detectors. For this system, Bohm [9] and Bell [6] both assume that the following highly idealized requirements hold:

(a) The initial state of the pair is a 100 % pure spin-singlet. (ψ = singlet = ↑↓—↓↑ in any coordinate system)
(b) Both particles enter the collimators.
(c) The system's collimation is perfect and the propagation is loss-free.
(d) The propagation and spin-state selection are depolarization-free, and
(e) Both detectors have 100 % efficiency.

It is important to note, in passing, that these idealized specifications are, in general, impossible to realize in practice.

We define the result values (see the section "Result Values and Expectation-Value Inequalities (E-Inequalities)") at each apparatus A $\equiv \pm 1$, and B $\equiv \pm 1$, respectively. With the above idealized specifications, for an ensemble of decaying spin-zero particles, the quantum mechanical predictions for the probabilities of the four possible outcomes are



$$\text{Q.M.} \Longrightarrow P(\vec{a}, \vec{b}) = \overline{AB} = \langle S=0 \,|\, \vec{\sigma}_1 \cdot \vec{a}\; \vec{\sigma}_2 \cdot \vec{b} \,|\, S=0 \rangle = -\vec{a} \cdot \vec{b}$$

Fig. 28.1 Bohm's [9] *Gedankenexperiment*, that provides the prototype for Bell's-Theorem discussions.

$$\begin{aligned}
&\mathrm{prob_{QM}}(A=1, B=1) = 1/2\sin^2(\mathrm{ang}(\mathbf{a},\mathbf{b})/2),\\
&\mathrm{prob_{QM}}(A=-1, B=-1) = 1/2\sin^2(\mathrm{ang}(\mathbf{a},\mathbf{b})/2),\\
&\mathrm{prob_{QM}}(A=1, B=-1) = 1/2\cos^2(\mathrm{ang}(\mathbf{a},\mathbf{b})/2),\\
&\mathrm{prob_{QM}}(A=-1, B=-1) = 1/2\cos^2(\mathrm{ang}(\mathbf{a},\mathbf{b})/2),
\end{aligned} \tag{1}$$

where ang(**a**, **b**) is the angle between the two Stern-Gerlach analyzer orientations, and $\mathrm{prob_{QM}}$ (A = i, B = j) is the probability (as predicted by quantum mechanics) that apparatus A will yield the result i, and that apparatus B will yield the result j.

It is the large amplitude sinusoidal dependence of Eqs. (1) that is at the heart of Bell's Theorem, and it was Bell's genius to first note that this dependence cannot be obtained by any local hidden-variables theory, but instead can only be obtained by quantum mechanics! Bell thus discovered that the large amplitude sinusoidal dependence in Eqs. (1) is strictly peculiar to quantum-mechanical entangled-state systems. He further discovered that virtually any reasonable attempt to model the behavior of Bohm's *Gedankenexperiment* via hidden-variables gave instead, a strange non-sinusoidal result, and/or a low amplitude result that is very different from that given by Eqs. (1).

Bell's observation thus became the inspiration for experimentalists, who, in turn, wondered if nature really behaves the way quantum mechanics strangely predicts here. Relaxation of the ideal specifications for this *Gedankenexperiment*, in turn, reduces the amplitude of this sinusoidal dependence, whereupon a normalization loophole can result when the relaxation goes too far. In practice, very little relaxation from the ideal can be tolerated.

One Possible Cause for the Normalization Loophole

An important but frequently overlooked feature of Bohm's *Gedankenexperiment* is that each apparatus provides the binary result, ± 1. Thus, for the two apparatuses and a given pair of spin-entangled particles, there are then only four possible outcomes, and four associated probabilities. For any set of probabilities to be sensible, and for Bell's Theorem to obtain, the sum of these four probabilities must be normalized to one. That is, we must have

$$\textstyle\sum_{i=\pm 1}\sum_{j=\pm 1} \mathrm{prob}(A(\mathbf{a})=i, B(\mathbf{b})=j) = 1, \text{ for all } \mathbf{a},\mathbf{b}. \tag{2}$$

We note here that this normalization condition holds for the quantum mechanical predictions of Eqs. (1).

One can measure the various prob(A(**a**) = i, B(**b**) = j) experimentally from event frequencies. To do so, one needs the total number of i, j events, N(A(**a**) = i, B (**b**) = j), normalized by the total number N of emitted-pair events. Then, if and only if all events are properly accounted for, the above normalization condition becomes

$$\sum_{i=\pm 1} \sum_{j=\pm 1} N(A(\mathbf{a}) = i, B(\mathbf{b}) = j) = N, \text{ for all } \mathbf{a}, \mathbf{b}. \quad (3)$$

We further note that if there are missing non-zero terms in these summations, then the normalization condition of Eq. (3) does not hold.

Now consider any actual realization of Bohm's *Gedankenexperiment*. There, we really have ternary-result rather than binary-result apparatuses. In practice, one or both of the particles will fail to enter the collimators. Additionally, any real detector will sometimes fail to detect a particle entering it, and/or will sometimes falsely detect a particle, even when one is not present (a "dark-count"). As a result, there will be un-paired detections at the two apparatuses and/or totally missing paired detections. Correspondingly, for any realization of Bohm's *Gedankenexperiment* we really have the possible outcomes for each apparatus as being one of three possibilities: +1, −1, and No-detection (with no result-value, as yet, assigned to this possibility). For binary-result apparatuses, there are 4 nonzero terms in the above double summations. For ternary-result apparatuses, however, there are 9 nonzero terms. Unfortunately, at most, only 8 of those 9 can be observed by the two apparatuses, since the 9th term is a probability of nothing happening at both apparatuses.

Of course the value of the 9th unobserved term can be determined via an *a priori* knowledge of N by using Eq. (3). This latter possibility is now commonly referred to as "heralding", wherein the source apparatus signals (heralds) that a particle pair has been emitted and is ready for analysis and detection. The possible use of said heralding measurements was first noted by Clauser and Shimony (CS) [15], and was therein given the name "event-ready detectors". (The modern term "heralding" had not yet been invented in 1978.) It is discussed in the section "CHSH R-Inequality with Heralding".

A simpler alternative to the use of heralding was offered by CH [13]. They avoid a need for knowing the value of N by producing a Bell Inequality that only involves ratios of the various N(A(**a**) = i, B(**b**) = j), whereupon the unknown value of N cancels out! Worries about unobserved particles may seem unimportant until one recognizes that in the earliest realizations of Bohm's *Gedankenexperiment*, the overwhelming majority of emitted pairs were, in fact, wholly or partially unobserved. The ratio of paired (coincident events) to unpaired detections ("singles events") detections was about 10^{-3}, while the ratio of the number of paired (coincident events) to the number of emitted particle pairs was about 10^{-6}. Only now, nearly 5 decades later have experiments evolved to the point where these event rates are all of comparable orders of magnitude.

Normalization-Loophole Free Clauser-Horne (CH) R-Inequality for Binary-Result Apparatuses

Clauser and Horne (CH) [13] start from an experimental arrangement that is slightly different from that of Fig, 28.1. It is shown in Fig. 28.2. It is configured to automatically enforce the above-noted need for binary results. A source at the center

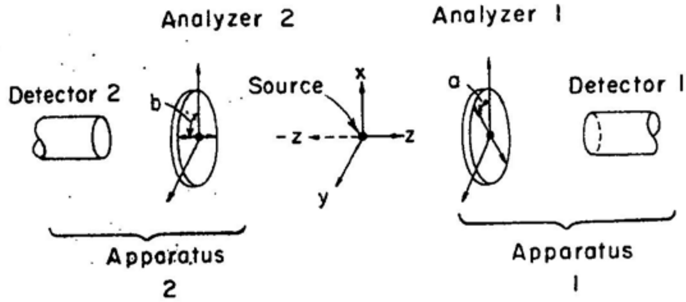


Fig. 28.2 Clauser and Horne's configuration for Bell's inequality experiments using binary-result apparatuses. Figure from Clauser and Shimony [14].

emits pairs of particles. The source is viewed by two apparatuses, named 1 and 2. Each apparatus consists of an adjustable attenuating analyzer and an associated single detector. A particle of the pair can pass through one of the analyzers, wherein after it is or is not detected by the associated detector. By the design of the experimental layout, only binary events, i.e. detection or non-detection events, can occur at each apparatus.

The CH derivation of Bell's Theorem and of the CH inequality starts by using directly observed event rates. In the apparatus of Fig. 28.2, one measures an individual detector's detection rate, and also simultaneously measures the ("coincident") paired detection rate of the two detectors. During a long period of time, t, the source emits say N of the two particle systems of interest. For this period, denote by $N_1(\mathbf{a})$ and $N_2(\mathbf{b})$ the number of detections at detectors 1 and 2 respectively, and by $N_{12}(\mathbf{a}, \mathbf{b})$ the number of nearly simultaneous (coincident) detections at the two detectors. From these numbers of detections, when sufficiently large, one may correspondingly define (measure) the ensemble probabilities

$$\begin{aligned} p_1(\mathbf{a}) &= N_1(\mathbf{a})/N, \\ p_2(\mathbf{b}) &= N_2(\mathbf{b})/N, \\ p_{12}(\mathbf{a},\mathbf{b}) &= N_{12}(\mathbf{a},\mathbf{b})/N. \end{aligned} \tag{4}$$

Here, p_{12} is the probability of joint (coincident) detections by both detectors; p_1 is the probability of a detection by detector 1, independently of what happens at detector 2; and p_2 is the probability of a detection by detector 2, independently of what happens at detector 1.

CH showed that the probabilities associated with correlated particle pairs that are described by any Local Realistic Theory (i.e. one that describes pairs of localized objects—see the Appendix), are constrained by the following inequality:

$$-1 \leq p_{12}(\mathbf{a},\mathbf{b}) - p_{12}(\mathbf{a},\mathbf{b'}) + p_{12}(\mathbf{a'},\mathbf{b}) + p_{12}(\mathbf{a'},\mathbf{b'}) - p_1(\mathbf{a'}) - p_2(\mathbf{b}) \leq 0. \tag{5}$$

The left-hand inequality requires careful normalization of these probabilities (i.e. it requires one to know N), but the right-hand inequality does not! It is independent of N. Via Eqs. (4) one then has

$$-N \le N_{12}(\mathbf{a},\mathbf{b}) - N_{12}(\mathbf{a},\mathbf{b'}) + N_{12}(\mathbf{a'},\mathbf{b}) + N_{12}(\mathbf{a'},\mathbf{b'}) - N_1(\mathbf{a'}) - N_2(\mathbf{b}) \le 0. \quad (6)$$

Dividing by t finally gives the associated CH R-inequality

$$-R_{Source} \le R_{12}(\mathbf{a},\mathbf{b}) - R_{12}(\mathbf{a},\mathbf{b'}) + R_{12}(\mathbf{a'},\mathbf{b}) + R_{12}(\mathbf{a'},\mathbf{b'}) - r_1(\mathbf{a'}) - r_2(\mathbf{b}) \le 0, \quad (7)$$

where, $R_{12}(\mathbf{a},\mathbf{b})$ is the coincidence detection rate for the two detectors, and $r_1(\mathbf{a}')$ and $r_2(\mathbf{b})$ are respectively the individual singles detection rate at detectors 1 and 2. The quantity R_{Source} is the source rate, which may be used if it has been measured via heralding. (See the section "CHSH R-Inequality with Heralding".) For experiments where a heralded source is not employed, one may rewrite the right-hand side of Ineq. (7) as

$$[R_{12}(\mathbf{a},\mathbf{b}) - R_{12}(\mathbf{a},\mathbf{b'}) + R_{12}(\mathbf{a'},\mathbf{b}) + R_{12}(\mathbf{a'},\mathbf{b'})]/[r_1(\mathbf{a'}) + r_2(\mathbf{b})] \le 1. \quad (7')$$

Here, the singles rates r_1 and r_2, are used to normalize the sum of four R_{12} coincidence rates. The minus sign preceding the second R_{12} term in the numerator may be permuted among any of the four terms and Ineq. (7') still holds.

Inequalities (5)–(7') are known as the CH inequalities. The CH R-inequality (7') is noteworthy in that it gets its normalization by using only the number of singles events at the two detectors. So doing, it provides a Bell Inequality that does not rely on the value of N (or R_{Source}), that is usually difficult to measure. Thereby Ineq. (7') is self-normalizing. The good news associated with the CH inequality is that the influence of N (or R_{Source}) vanishes, and the inequality is normalization-loopholefree. The bad news is that from an experimental viewpoint, the CH inequality is very difficult to violate. For low detector efficiencies and/or for small solid-angle collection efficiencies, the singles rates r_1 and r_2 are typically very much larger than R_{12}, (by a factor of about 10^3 for the typical cascade-photon experiments), Ineq. (7') is then automatically satisfied, whereupon no normalization-loophole free experiment can then be done.

For these (and for all other Bell Inequalities), it is generally necessary to perform a sequence of different experiments and compare their different results. In the present case there are four required experiments, at each of the respective analyzer orientation pairs, (**a**,**b**), (**a**,**b'**), (**a'**,**b**), and (**a'**,**b'**). Additionally, measurements must also be taken of the normalizing singles count rates r_1 and r_2 at angles **a'** and **b**, respectively, although these measurements are usually already obtained simultaneously during the coincidence rate measurements.

The experimental difficulties associated with designing an experiment to violate Ineq. (7') were lessened significantly by an observation made by Eberhard [17] in 1993. When the quantum state of the particle pair is maximally entangled, the singles rates $r_1(\mathbf{a}')$ and $r_2(\mathbf{b})$ in Ineq. (7') are constant, (i.e. independent of **a**' and **b**). On the other hand, Eberhard noted that there is no need for the normalizing denominator in Ineq. (7') to be constant. If the quantum state of the particle pair is not maximally entangled, then at least one of these two singles rates may be made small, and Ineq. (7') is more readily violated. Eberhard's observation has led to the recent experimentally observed violations of the CH inequality by Giustina et al. [24] and by Christensen et al. [11].

Clauser and Shimony [15] show in their review article that the methods of proof of Bell's Theorem used by Wigner [41], Bell [7], Belinfante [5], and Holt [26] can all lead to the CH inequality. CS further note that the CHSH inequality is a special case of the CH inequality, and that Bell's 1964 inequality [6] in turn is a special case of the CHSH inequality.

Result Values and Expectation-Value Inequalities (E-Inequalities)

Bell's original 1964 inequality and the CHSH inequality are both in a form that constrains expectation values for observed "result values". Herein, we refer to these as E-inequalities. The expectation values are calculated using previously defined "result values". What is a "result value", and why is it needed? To some extent, for Bell's Theorem, result values are only a historical artifact, and, as noted above in the section "Normalization-Loophole Free Clauser-Horne (CH) R-Inequality for Binary-Result Apparatuses", Bell Inequalities can be derived without invoking them. One may ask, where did they come from?

Bohm was discussing the quantum theory of the measurement process and the Einstein, Podolsky, and Rosen [19] paradox when he introduced his 1951 *Gedankenexperiment* of Fig. 28.1. In such a discussion, it is presumably necessary to assert that something is actually being "measured". Whatever is being measured, then should have a "result value" that is to be determined by the "measurement". The assumptions underlying Bell's Theorem, on the other hand, do not depend on whether or not something is being measured. In fully general LHVT's or Local Realistic Theories, one really does not have the faintest idea about what one is doing on a microscopic level when one performs a given experiment. Indeed, a dispute over what is happening internally in such an experiment (or indeed, if anything is happening at all on a microscopic level, as Bohr insisted) is at the very heart of any fully general theory that is an alternative to or consistent with quantum mechanics. Recall that the Copenhagen interpretation of quantum mechanics asserts that there is no possible explanation of the microphysics of Bohm's *Gedankenexperiment*, whereupon it would seem to be highly presumptuous to assert that one knows what one is measuring! Correspondingly, for a discussion of Bell's Theorem, the result values $A, B \equiv \pm 1$ chosen by Bohm and Bell are perfectly arbitrary. Mostly, they simply provide names like Tom, Dick, Harry, +1, 0, spin-up, top, beauty, and $+\hbar/2$, etc. for the possible experimental outcomes. Indeed, the CH inequality which governs Local Realism dispenses with the use of result values altogether!

For a discussion of Bell's Theorem, however, CHSH and Bell did find some utility in defining result-values. If the result values have integer numerical values, then they may be used as indices in summations. Result values also allow one to construct expectation values for these results, which then lead to E-inequalities. Additionally, expectation values are readily calculated using quantum mechanics via matrix elements. Then, as long as care is exercised in converting said E-inequality into a useful R-inequality, result values serve their purpose.

The expectation value of the product of the binary-result values for A and B is also known as the correlation function of these two values. For Bohm's 1951 *Gedankenexperiment*, it is given by

$$E(\mathbf{a},\mathbf{b}) \equiv \langle AB \rangle = \sum_{i,j=\pm 1} A(\mathbf{a})B(\mathbf{b})\ \mathrm{prob}(A(\mathbf{a})=i, B(\mathbf{b})=j). \tag{8}$$

Using Eqs. (1), one can calculate the quantum mechanical prediction for this correlation function for Bohm's idealized *Gedankenexperiment* of Fig. 28.1 as

$$E_{QM}(\mathbf{a},\mathbf{b}) = -\mathbf{a}\cdot\mathbf{b}. \tag{9}$$

In order to measure $E(\mathbf{a}, \mathbf{b})$ for use in the CHSH inequality, one must measure the various $\mathrm{prob}(A(\mathbf{a}) = i, B(\mathbf{b}) = j)$, for all i and j in a ternary result experiment, i.e. a realization of Bohm's 1951 *Gedankenexperiment*. To do so using event frequencies, one needs (for sufficiently large N)

$$\mathrm{prob}(A(\mathbf{a})=i, B(\mathbf{b})=j) = N(A(\mathbf{a})=i, B(\mathbf{b})=j)/\sum_{\mathrm{all}\ i}\sum_{\mathrm{all}\ j} N(A(\mathbf{a})=i, B(\mathbf{b})=j) \tag{10}$$

where the double summation in the denominator must be taken over all possible values of i and j.

As we have noted above, this cannot be done without a knowledge of N. The possible routes are then

1. Ignore details of a realization of Bohm's *Gedankenexperiment*, and thereby ignore the normalization loophole, as was done by Bell [6], 1964 (See the section "Bell's 1964 E-Inequality for Idealized Binary Result Apparatuses").
2. Use the CH experimental configuration of Fig. 28.2 and define "Detection" and "NoDetection" to be the binary results needed. This option only works with very high detection efficiency, and was used first experimentally by Giustina et al. [24] and by Christensen et al. [11].
3. Use the Bell experimental configuration described in the section "CHSH R-Inequality With Heralding" and measure N via heralding, as was done experimentally by Rowe et al. [35] and by Matsukevitch et al. [33].

4. Use the CH experimental configuration of Fig. 28.2 and the polarizer-removal protocol given by CHSH, along with the associated CH supplementary assumption. This option was used by the earliest Bell's Inequality experimental tests, the first of which was by Freedman and Clauser [20].
5. Bypass the normalization loophole by employing a supplementary assumption that is much stronger than that needed by the polarizer-removal protocol. This method is herein referred to as GR/AGR normalization (See the section "Garuccio and Rapisarda/Aspect Grangier Roger R-Inequalities for Real Ternary-Result Apparatuses"). It was first used experimentally by Aspect, Grangier and Roger (AGR) [3] in 1982.

Bell's 1964 E-Inequality for Idealized Binary Result Apparatuses

Bell's original 1964 paper [6] considered Bohm's idealized *Gedankenexperiment*, and assumed the idealized specifications as listed above in the section "Bohm's 1951 *Gedankenexperiment* and Its Relation to Bell's Theorem". Given these specifications, he noted (indeed he required) that the quantum mechanical prediction for parallel analyzers has the value

$$E(\mathbf{a},\mathbf{a}) = -1. \tag{11}$$

Bell assumed Eq. (11) to hold exactly for at least one value of **a**. This assumption thus requires that for that value of **a**, the *Gedankenexperiment* must exhibit a perfect correlation. Using this assumption, he first notes that determinism follows directly from it for the *Gedankenexperiment*. In addition, using these assumptions, he goes on to show that no Local Hidden Variables Theory (LHVT) can give the quantum mechanical prediction for Bohm's idealized *Gedankenexperiment*. He does so by showing that the inequality

$$1 + E_{LHVT}(\mathbf{b},\mathbf{c}) \geq |E_{LHVT}(\mathbf{a},\mathbf{b}) - E_{LHVT}(\mathbf{a},\mathbf{c})|, \tag{12}$$

holds for any LHVT. Surprisingly, he discovers that the quantum-mechanical prediction for this system given by Eq. (9) does not satisfy Ineq. (12) for a significant range of **a** and **b**.

Unfortunately, Bell's mathematical analysis applies only to totally idealized systems, as per the discussion of the section "Bohm's 1951 *Gedankenexperiment* and Its Relation to Bell's Theorem", above. With even an infinitesimal departure from the perfect system described in that section, his mathematical arguments fail.

Here, it is perhaps appropriate to invoke Ben Franklin's famous observation:

"The only predictions that can be made with certainty are for death and taxes."

From a physicist's standpoint, Ben Franklin's observation can be paraphrased (interpreted) to mean

> If a theoretical argument relies on predictions that must obtain with certainty, but said predictions can never obtain in reality, then that argument applies only to a vanishing subset of reality that never occurs, whereupon the argument applies to nothing at all, and thus may be disregarded.

CHSH noted that Bell's argument requires Eq. (11) to hold exactly for at least one value of **a**. However, since Eq. (11) is not equivalent to either death or taxes, then it never obtains for any realizable (or real) systems. Correspondingly, Bell's argument (as it stands) does not apply to any realizable (or real) systems. Unfortunately, without the constraint by Eq. (11), for at least one value of **a**, Bell's mathematical argument fails. Fortunately, despite the above paraphrasing of Ben Franklin, Bell's mathematical reliance on Eq. (11) by no means implies that Bell's result may be disregarded. CHSH show that Eq. (11) is not a necessary requirement for a useful (but different) Bell Inequality to be derived via an alternative mathematical argument.

Clauser Horne Shimony Holt (CHSH) E-Inequality for Real Binary Result Apparatuses

CHSH first showed that for systems that do not comply with the unrealizable specifications outlined in the section "Bohm's 1951 *Gedankenexperiment* and Its Relation to Bell's Theorem" for Bohm's *Gedankenexperiment*, and especially for systems that do not comply with the unrealizable restriction of Eq. (11), then an alternative inequality can be written that does apply to realizable systems. CHSH show that all deterministic local hidden variable theories are constrained by the alternative E-inequality for real binary result apparatuses,

$$|E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{c})| \leq 2 - E(\mathbf{b'},\mathbf{b}) - E(\mathbf{b'},\mathbf{c}). \qquad (13)$$

Shimony [37] further pointed out that Ineq. (13) can be rewritten as

$$-2 \leq E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b'}) + E(\mathbf{a'},\mathbf{b}) + E(\mathbf{a'},\mathbf{b'}) \leq 2, \qquad (14)$$

wherein the single minus sign may be permuted among the four terms. Unlike Bell's original inequality (12), the CHSH Ineq. (14) applies to realizable systems. The LHVT subscript has been dropped because CH subsequently showed that Ineq. (14) also holds for the more general theories of Local Realism.

Clauser Horne Shimony Holt (CHSH) R-Inequalities for Real Binary-Result Apparatuses via the CHSH Polarizer-Removal Protocol

Since Bell [6] did not address either the experimental status or the testability of his results, Clauser Horne Shimony Holt (CHSH) [12] in 1969 pursued those issues. Armed with Ineq. (14), they first noted that there were no existing experimental data available for comparison with Ineq. (14). They then proposed a real experiment to fill this gap and to actually test Ineq. (14). Their proposed experiment was for a modification of an experiment that had been performed two years earlier by Kocher and Commins [28].

In the CHSH-proposed experiment, two polarization-entangled photons are emitted by an atomic-cascade decay. High efficiency linear polarizers are then used to analyze the entangled photons. These polarizers then replace the Stern Gerlach analyzers of Bohm's *Gedankenexperiment*. To maintain binary-result apparatuses, CHSH use the apparatus configuration of Fig. 28.2. They first propose using the associated result values Detection = +1, NoDetection = −1. Unfortunately, with the technology and detector efficiencies available in 1969, it was still not possible to violate Ineq. (14) using those definitions. Undeterred, CHSH introduce the polarizer-removal protocol. Under this protocol, coincidence rates are measured with both polarizers in place as a function of the polarizer orientations. Additionally, coincidence rates are measured with one polarizer, or the other, or both polarizers removed. The needed multiplicity of experiments, i.e. experiments with polarizer(s) removed and with polarizers inserted at the various needed orientations, are all performed in such a manner that the source rate and the effective detector acceptance solid-angles remain unchanged among them. CHSH modify the result definitions from Detection/ No Detection to Passage/ NoPassage of the photons through the polarizers. CHSH then offer the following supplementary assumption:

> Given the emergence of a pair of photons from the associated pair of analyzers, we assume that the joint detection probability is independent of the analyzer orientations ***a*** *and* ***b***.

Using the polarizer-removal protocol and the CHSH (or CH—see below) supplementary assumption, one can then write for the correlation function

$$E(\mathbf{a},\mathbf{b}) = 1 + 4\,[R(\mathbf{a},\mathbf{b}) - 2R(\mathbf{a},\infty) - 2R(\infty,\mathbf{b})]/R(\infty,\infty), \tag{15}$$

where the symbol, ∞, denotes the exceptional case when a polarizer has been removed. The associated measured coincidence detection rates may be abbreviated as

$$\begin{aligned} R(\mathbf{a},\infty) &\equiv R_1(\mathbf{a}),\\ R(\infty,\mathbf{b}) &\equiv R_2(\mathbf{b}),\\ R(\infty,\infty) &\equiv R_0. \end{aligned} \tag{16}$$

Further, assuming that $R_1(\mathbf{a}) = R_1$, and that $R_2(\mathbf{b}) = R_2$ are both measured to be constant and respectively independent of **a** and **b**, then CHSH show that (13)–(16) can be combined to yield the CHSH R-inequality

$$|R(\mathbf{a},\mathbf{b}) - R(\mathbf{a},\mathbf{c})| + R(\mathbf{b'},\mathbf{b}) + R(\mathbf{b'},\mathbf{c}) - \mathbf{R_1} - \mathbf{R_2} \leq \mathbf{0}. \quad (17)$$

Also, when **a** and **b** are the scalar angles a and b, as shown in Fig. 28.2, and when it is experimentally demonstrated that the coincidence rate R(**a**,**b**) only depends on the angle $\Phi \equiv \text{ang}(\mathbf{a},\mathbf{b})$ between the polarizers, as per

$$R(\mathbf{a},\mathbf{b}) = R(\Phi), \quad (18)$$

then Ineq. (17) can be written as

$$-R_0 \leq 3R(\Phi) - R(3\Phi) - R_1 - R_2 \leq 0. \quad (19)$$

Freedman [18] further noted that if one takes the optimal value $\Phi = \pi/8$ for maximal violation of (19) by cascade-photon experiments, then a particularly compact form of a Bell R-inequality results, as per

$$|R(\pi/8) - R(3\pi/8)|/R_0 \leq 1/4. \quad (20)$$

Here, the coincidence rate with polarizers in place, R (Φ) is normalized by the coincidence rate with polarizers removed, R_0. Significant utility is provided by this compact form in that only three independent coincidence rates need be measured for it to be tested, although it is still necessary to verify the required rotational invariance of R(Φ). Such rotational invariance can be assured, however, by simply averaging the R(Φ) measurement over common rotations of the pair of analyzers.

Freedman's version of the CHSH inequality is noteworthy in that it provides a somewhat graphic measure of the minimum sinusoidal amplitude variation of the normalized coincidence rate that is needed to violate a Bell Inequality. It also graphically indicates that if R(Φ) in Ineq. (20) is normalized by a coincidence rate other than R_0, say by a smaller rate, then a larger violation occurs, and a violation may then occur where it otherwise would not. On the other hand, if the magnitude of R(Φ) is significantly diminished, say by even modest absorption by the polarizers, then no violation of Ineq. (20) can occur, and the experimental configuration is insufficient to test a Bell Inequality. Thus, violation or no-violation of the CHSH inequality critically depends on the count-rate's normalization.

The experimental requirements for a violation of Ineqs. (17), (19) or (20) are highly demanding upon the required polarizer quality.. Those requirements are specified quantitatively by CHSH for their proposed experiment. That experiment was first performed by Freedman and Clauser in [20]. They found that the only available polarizers (in 1972) meeting the requirements for very low absorption were the pile-of-plates variety. Most other early experiments testing the CHSH inequality followed their example, and also used pile-of-plates polarizers.

Clauser and Horne (CH) [13] significantly improve upon the CHSH supplementary assumption. They thus provide a much weaker supplementary assumption that leads to the same result as that by CHSH. CH call their improved supplementary assumption the "no-enhancement" assumption. It is the following:

> We assume that the presence of an analyzer does not somehow enhance (increase) a particle's probability of detection, relative to the probability of its detection with the polarizer absent.

Under the CH no-enhancement assumption, and with the polarizer-removal protocol, the CH Ineq. (5), discussed in the section "Normalization-Loophole Free Clauser-Horne (CH) R-Inequality for Binary-Result Apparatuses" above, reduces to the CHSH R-inequality predictions Ineqs. (17)–(20), whereupon the Freedman-Clauser experiment [20] refutes Local Realism (but is still, of course, subject to the locality loophole).

The CH Counterexample

Clauser and Horne [13], provide an *ad hoc* counterexample that employs enhancement and that can predict the Freedman-Clauser [20] results. Thus, despite the high plausibility of the associated CH supplementary assumption, their counterexample shows that the no-enhancement assumption (or some other assumption) is still needed for experiments that use the polarizer–removal protocol in order to evade the normalization loophole. Under the CH counterexample, the normalization loophole is carefully exploited, in a somewhat pathological manner. Low detection efficiency may be present in the system for a variety of reasons. The CH counterexample carefully exploits the low efficiency produced by absorbing polarizers and other losses to "collude" with the detectors to generate an anomalous violation of the CHSH R-inequality by a Local Realistic theory. Under the CH counterexample, some photons passing through a polarizer have diminished detection probability, i.e. their detection probability is attenuated. Other photons passing through the polarizer have increased detection probability, i.e. their detection probability is "enhanced", or supercharged. Recall that the CH supplementary assumption (see the section "Clauser Horne Shimony Holt (CHSH) R-Inequalities for Real Binary-Result Apparatuses via the CHSH Polarizer-Removal Protocol") specifies that this latter enhancement process does not occur. In the CH counterexample, the polarizer and detector collude with each other to create an anomalous violation. While such "collusion" seems pathological, it should be noted that similar collusion was taken seriously when one considered tests of the locality loophole, as mentioned above. A major difference here is that the malevolent force providing the collusion, as mentioned above, now must be nature, herself, rather than that by a determined cryptography eavesdropper.

Marshal et al. [31] offer a counterexample that is vaguely similar to the CH counterexample. While the CH counterexample generates an exactly sinusoidal variation of the coincidence rate as a function of polarizer orientation, the Marshal-Santos-Selleri counterexample generates a non-sinusoidal variation that would have been readily apparent in the experimental data, which they claim "*fits the existing data as closely as the quantum-mechanical model.*" Scrutiny of their model's prediction, however, reveals their claim to be false, at least for the "existing data" from the Freedman Clauser [20] experiment.

CHSH R-Inequality with Heralding

Bell in his 1971 paper [7] continued to use Bohm's ternary-result *Gedankenexperiment* that he used in his 1964 paper [6]. In this second paper, he acknowledged the CHSH assertion that one must account for the failure of one or both of the apparatuses to detect a particle. To handle this situation, he first showed that the CHSH E-inequality obtains as long as the result-values A and B are defined such that $|A| \leq 1$ and $|B| \leq 1$. Correspondingly, he proposed the definitions,

$$A, B \equiv \pm 1 \text{ Detections, and } 0 \equiv \text{No Detection,} \tag{20}$$

for use with ternary-result apparatuses. Clauser and Horne [13], in their Appendix B, however, show that Bell's scheme will not work in an actual experimental context, because it requires measuring events where nothing happens. It eventually became clear (private communication between CH and Bell) that Bell was tacitly assuming that "it was already known (by some unspecified means) that a particle pair was emitted into the associated detector entrance solid angles", whereupon accountability of the unobserved events is then possible. Clauser and Shimony [15] thus clarified Bell's [7] proposed scheme by depicting "Bell's configuration", as shown in Fig. 28.3, and contrasting it with the "CH configuration" of

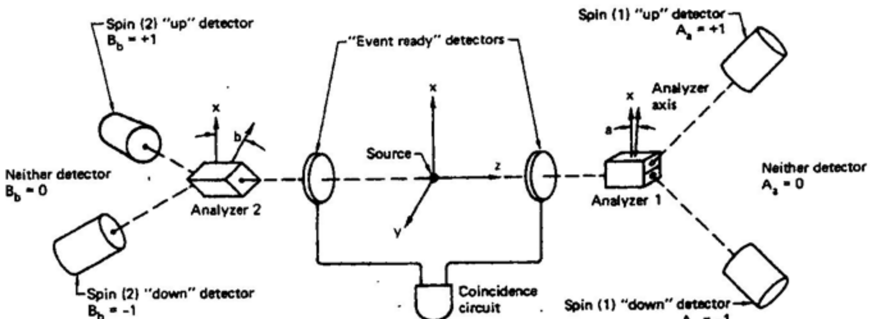


Fig. 28.3 Bell' configuration for Bell's inequality experiments using ternary-result apparatuses and source heralding. Figure from Clauser and Shimony [14].

Fig. 28.2. Thus explicitly clarified, Bell's configuration includes "event-ready" detectors as a means for specifically knowing that a particle pair has been emitted into the associated detector entrance solid angles. Subsequently, the term "heralding" has been coined to describe this process.

In order to use heralding and Bell's [7] result-value definitions in the CHSH E-inequality (14) with an experiment that uses ternary-result apparatuses, one may rewrite Eq. (8) for ternary-result apparatuses as

$$E(\mathbf{a},\mathbf{b}) \equiv \ <AB> \ = \sum_{i=-1,0,1} \sum_{j=-1,0,1} A(\mathbf{a})B(\mathbf{b})\ \mathrm{prob}(A(\mathbf{a})=i, B(\mathbf{b})=j). \tag{21}$$

The probabilities appearing in Eq. (21) may now be measured using observed count rates via

$$\mathrm{prob}(A(\mathbf{a})=i, B(\mathbf{b})=j) = R(A(\mathbf{a})=i, B(\mathbf{b})=j)/R_{\mathrm{Source}}, \tag{22}$$

where R_{Source} is the source rate, as measured via heralding. To be sure, some of the needed detection rates in Eqs. (21) and (22) are unmeasured, i.e. those for $i = 0$ and/or $j = 0$, and thus are not known. However, since their contributions to Eq. (21) have zero for their associated coefficients in Eq. (21), their unknown values are of no importance. The expectation values needed for the CHSH E-inequality (14) are now fully determined by observed detection rates, and are given by

$$E(\mathbf{a},\mathbf{b}) = [R_{+,+}(\mathbf{a},\mathbf{b}) + R_{-,-}(\mathbf{a},\mathbf{b}) - R_{+,-}(\mathbf{a},\mathbf{b}) - R_{-,-}(\mathbf{a},\mathbf{b})]/R_{\mathrm{Source}}, \tag{23}$$

where the following shorthand notation is used: $R_{i,j} \equiv R(A(\mathbf{a}) = i, B(\mathbf{b}) = j)$.

Garuccio and Rapisarda/Aspect Grangier Roger R-Inequalities for Real Ternary-Result Apparatuses

Garuccio and Rapisarda (GR) [22] in 1981 proposed a new method for normalizing coincidence rates for "ternary-result" apparatuses without the associated requirement for heralding. Thereby they provide a new R-inequality, that was first tested experimentally by Aspect, Grangier, Roger (AGR) [3] in 1982. These efforts proceeded despite known difficulties for ternary-result apparatuses, as found earlier by CH [13] and by CS [16]. Recall that Bell had originally proposed the use of ternary-result apparatuses, first, inadvertently, in 1964, and again, advertently, in 1971 [7] (see the section "CHSH R-Inequality with Heralding"). In his 1971 paper, Bell thus proposed using the ternary-result values (+1, −1, and 0), wherein 0 represents unobserved No-Detection events in the CHSH E-inequality. Unfortunately, he did not offer an associated testable R-inequality. He tacitly assumed that source heralding was being used, although he did not specifically say so. However, CH [13] (see their Appendix B) reanalyzed Bell's 1971 proposal [7] and found that, no loophole-free testable R-inequality can be generated for such systems, because they require the measurement of a probability of nothing happening, which cannot be done without knowing the associated probability of something happening. Knowing the latter then requires knowing the source rate, R_{Source}, (e.g. via a heralded source). CH show that, alternatively, one may convert each ternary-result apparatus into a binary-result apparatus, "at the very beginning of the derivation", and then use the CH inequality. CH pointed out that such conversion can be accomplished, for example, by combining the No-Detection "0" channels with say the −1 Detection channels at each apparatus, and then by looking only at the +1 Detection channels. Such a conversion is tantamount to simply ignoring the −1 detections. CH show that by doing so, one can then generate a CH inequality (using Bell's method of proof) that involves only (+1,+1) detections. Actually, depending upon which pair of channels one chooses to ignore, one can alternatively generate four independent CH inequalities for each of the (+1,+1), (−1, −1), (−1,+1), and (+1, −1) channels. Of course, such a conversion destroys any symmetry between the (+1,+1) channels and the partly unobserved (−1 & 0, −1 & 0) channels. But, of course, neither locality, nor realism, nor quantum mechanics have any need or requirement for experiment symmetry.

CH inequalities generated by ignoring channels were still not testable using the technology available in 1982, i.e. by using low quantum-efficiency photomultiplier tube detectors and atomic-cascade decay entangled-photon sources. With 1982 technology, some additional experimental protocol and set of supplementary assumptions was thus still needed for testing ternary-result apparatus experiments.

Despite these known difficulties for ternary-result apparatus experiments, Aspect, Grangier, Roger (AGR) [3] in 1982 attempted to build a ternary-result apparatus for testing a Bell Inequality by using a supplementary assumption proposed earlier by Garuccio and Rapisarda [22]. As a starting point, GR/AGR use the CHSH E-inequality (14),

$$-2 \leq E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}') + E(\mathbf{a}',\mathbf{b}) + E(\mathbf{a}',\mathbf{b}') \leq 2. \tag{14}$$

Their experiment uses the Bell's configuration of Fig. 28.2, except that it does not include the "event-ready detectors" used for heralding by that configuration.

In the section "One Possible Cause for the Normalization Loophole" above, we note that a ternary-result apparatus experiment requires one to know the values of nine independent coincidence rates, wherein their sum can be used for probability normalization. In the AGR experiment, however, only four coincidence rates are measured. Using Bell's [7] ternary-result values, (+1, −1, and 0), where 0 represents unobserved No-Detection events, the four coincidence rates measured by AGR are

$R_{++}(\mathbf{a}, \mathbf{b}) \equiv (A = +1, B = +1)$ observed coincidence rate,
$R_{--}(\mathbf{a}, \mathbf{b}) \equiv (A = -1, B = -1)$ observed coincidence rate,
$R_{+-}(\mathbf{a}, \mathbf{b}) \equiv (A = +1, B = -1)$ observed coincidence rate,
$R_{-+}(\mathbf{a}, \mathbf{b}) \equiv (A = -1, B = +1)$ observed coincidence rate.

The other five (needed) coincidence rates that AGR do not measure are:

$R_{+0}(\mathbf{a}, \mathbf{b}) \equiv (A = +1, B = 0)$ unobserved "coincidence" rate,
$R_{0+}(\mathbf{a}, \mathbf{b}) \equiv (A = 0, B = +1)$ unobserved "coincidence" rate,
$R_{-0}(\mathbf{a}, \mathbf{b}) \equiv (A = -1, B = 0)$ unobserved "coincidence" rate,
$R_{0-}(\mathbf{a}, \mathbf{b}) \equiv (A = 0, B = -1)$ unobserved "coincidence" rate,
$R_{00}(\mathbf{a}, \mathbf{b}) \equiv (A = 0, B = 0)$ unobserved "coincidence" rate.

While the first four of these last five could, in fact, have been determined from the singles rates, that last one of these five, i.e. the (NoDetection, NoDetection) coincidence rate cannot be measured.

As per the discussion of the section "Result Values and Expectation-Value Inequalities (E-Inequalities)" above, the appropriate expectation value for use in the CHSH E-inequality (14) is

$$E_{CHSH}(\mathbf{a},\mathbf{b}) = \frac{R_{++} + R_{--} - R_{+-} - R_{-+} + 0 \times [R_{+0} + R_{-0} + R_{0+} + R_{0-} + R_{00}]}{R_{++} + R_{--} + R_{+-} + R_{-+} + [R_{+0} + R_{-0} + R_{0+} + R_{0-} + R_{00}]}$$
$$= \frac{R_{++} + R_{--} - R_{+-} - R_{-+}}{R_{++} + R_{--} + R_{+-} + R_{-+} + [R_{+0} + R_{-0} + R_{0+} + R_{0-} + R_{00}]}. \quad (24)$$

To make their results testable, AGR use the GR normalization scheme and arbitrarily set equal to zero the term in square brackets in the normalizing denominator that includes the five unobserved rates. So doing, they define

$$E_{GR/AGR}(\mathbf{a},\mathbf{b}) \equiv \frac{R_{++}(\mathbf{a},\mathbf{b}) + R_{--}(\mathbf{a},\mathbf{b}) - R_{+-}(\mathbf{a},\mathbf{b}) - R_{-+}(\mathbf{a},\mathbf{b})}{R_{++}(\mathbf{a},\mathbf{b}) + R_{--}(\mathbf{a},\mathbf{b}) + R_{+-}(\mathbf{a},\mathbf{b}) + R_{-+}(\mathbf{a},\mathbf{b})}. \quad (25)$$

The CHSH E-inequality (14) and GR/AGR's definition (25) are then combined to produce a new GR/AGR R-inequality, which was then violated by their experimental data.

We note, however, that the deleted term in square brackets in the denominator of Eq. (24) is very much larger than the remaining terms by a factor of about 10^6, when it is used with an experiment that uses an atomic cascade photon source and photomultiplier tube detectors, as did the AGR [3] experiment. Correspondingly, omission of that very large term by GR/AGR deserves careful scrutiny, especially since it represents a dramatic renormalization (by a factor of about 10^6). Without the term's deletion, it would be impossible for $E_{GR/AGR}(\mathbf{a},\mathbf{b})$ as given by Eq. (25), to violate (14).

GR and AGR state that

> ...we assume that the ensemble of actually detected pairs is a faithful example of all emitted pairs. ....

The GR/AGR assumption is now commonly and "gratuitously" referred to as the "fair-sampling assumption". This author's use of the description "gratuitous" will now be justified.

Let us examine the implications of GR/AGR's so-called "fair-sampling assumption". An important issue is whether or not it is consistent with the requirements and/or the assumptions underlying either quantum mechanics and Local Realism. In particular, one must first consider whether or not this supplementary assumption tacitlyor explicitly assumes/requires that all detected events have the same *a priori* probability of detection.

The GR/AGR assumption takes a particle-centric view of wave-particle duality / wave-particle ambiguity, which asserts an equivalence of the wave and particle viewpoints in quantum mechanics, but leaves vague and/or ambiguous the mechanism providing said equivalence. In a wave-centric viewpoint, the inverse square law for detected flux versus source-to-detector distance is due to diminished wave amplitude with increasing distance, and an associated diminished detection probability with said diminished amplitude. However, in a particle-centric view, it is due instead to a diminished geometrically diluted particle flux. In a particle centric view, particle detection probability is always constant, given a particle's presence at a detector. On the other hand, in a wave-centric view, particle detection probability is variable and depends on wave amplitude. Wave amplitude at each detector, in turn, may depend on the associated analyzer's orientation.

Consider first an assumption underlying quantum mechanics. Under Born's rule for calculating probabilities, a particle's detection probability is proportional to the absolute square of its probability amplitude. Particles that pass through polarization analyzers at differing orientations will have different transmitted probability amplitudes. Correspondingly, not all particles arriving at a detector will have the same probability amplitude or the same a priori detection probability. Thus, under a fundamental requirement by quantum mechanics, different events must be allowed to have different *a priori* detection probabilities. Any reasonable supplementary assumption used for a Bell's Inequality test must correspondingly allow for and be consistent with this possibility. GR/AGR instead assume particle detection probability is always constant, and *a fortiori* exclude theories with variable detection probability (including quantum mechanics).

Next consider a straightforward local realistic theory in which a photon is modeled simply as a short-pulse (or wave packet) of classical electromagnetic radiation. Under this theory, for example, one may assume that the semiclassical model for the photoelectric effect proposed by Lamb and Scully [30] holds. Then a pulse with a large classical amplitude will have a higher probability of generating a photoelectron, and an associated detectable output pulse of electric current from a photomultiplier tube, than will one with a small classical amplitude. Again, under Local Realism, different events must be allowed to have different *a priori* detection probabilities.

We thus see that properly testing both quantum mechanics and Local Realism requires one to allow for a variable detection probability of the detected particles by the particle detectors. Now, consider the implications of this requirement. Naturally, the set of detected particle events will have a preponderance of events with a higher *a priori* detection probability than will the set of undetected-particle events, which, in turn, will be dominated by events with a lower *a priori* detection probability. Correspondingly, the ensemble of detected pairs is clearly not "a faithful example of all emitted pairs" as GR/AGR assert. Instead, it provides an ensemble that is significantly biased in favor of events with a high *a priori* detection probability.

One may view the detection process as a competition for detection among the particles at the detectors. The GR/AGR assumption implies that the winners (the detected particles) in the competition for detection were no more capable of "winning the competition" (being detected) than were the losers (the undetected particles) in the competition. Viewed in this light, one may ask, is the GR/AGR assumption truly reasonable, and is the sampling truly "fair"? For comparison, note that in atmost any sports competition, it would be hard to find a winner who didn't truly believe that he/she did not "fairly" win the event because of superior ability (i.e. a higher *a priori* probability of winning) rather than by simple luck. Such a sports competitor would thus strongly disagree with the GR/AGR assumption as being "fair" and reasonable. Correspondingly, an assumption that all of the "winners" in a competition for being detected by a photomultiplier tube were equally capable of winning the competition, is equivalent to saying that the detected particles form a representative subset of all of the emitted particles, as far as their probability for being detected is concerned.

The GR/AGR assumption is thus a very strong assumption, indeed! It even seems to violate the fundamental premises underlying both Local Realism and quantum mechanics. Correspondingly, its application to the testing of not only the above very reasonable Local Realistic model, but also to the testing of quantum mechanics itself appears to be highly dubious. Clearly then, the appellation "fair sampling assumption" must be considered gratuitous.

It should also be noted that the GR/AGR assumption is not equivalent to the very much weaker CH no-enhancement assumption, as coupled to the CHSH polarizer-removal protocol. By contrast, the expectation that different particles may have different *a priori* detection probabilities is explicit in the CH no-enhancement assumption. CH simply assume that passage of a photon through a (presumably) attenuating polarizer does not somehow enhance its *a priori* detection probability. Moreover, polarizer absorption has a very strong effect when one is using the CHSH R-inequality (17), (19) or (20) that results from the CHSH polarizer-removal protocol and the no-enhancement assumption. In such a case, when the polarizer absorption is even modestly too large, no inequality violation occurs. It also has a very strong effect on the viability of the CH counterexample. On the other hand, photon absorption by a polarizer has no effect at all on the resulting numerical value obtained from using Eq. (25) in (14), and correspondingly has no effect on whether or not a violation occurs.[2]

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[2]In defense of the AGR experiment's polarizer parameters, their parameters do appear to meet the CHSH transmission requirements, although they are not required to do so in order to violate the

As suggested in the introduction, in order to evaluate a supplementary assumption, one may also compare how reasonable it is with how contrived an associated counterexample is. Given the GR/AGR normalization method's relative insensitivity to polarizer absorption, it is not surprising that one can readily build counterexamples that do not violate the CHSH E-inequality when the polarizer-removal protocol and no-enhancement assumption is used, but do violate the CHSH E-inequality when the GR/AGR assumption and protocol are used. (See Clauser [16].)

Further evidence of just how vulnerable the GR/AGR assumption is to counterexamples was given by Gerhardt et al. [23], who provide both a theoretical and a convincing experimental demonstration of the ease by which an actual experiment can be countered, especially in "security related scenarios". It should be noted that Gerhardt et al.'s demonstrated violations of a Bell Inequality all use GR/AGR normalization. Gerhardt et al., however, mistakenly attribute their counterexample's existence to a loophole in what they mistakenly refer to as the CHSH inequality, which instead is really the GR/AGR R-inequality produced by a combining (14) and (25) above. It should be emphatically noted that Gerhardt et al. do not produce an experimental (or theoretical) counterexample that employs the CHSH polarizer-removal protocol. However, they do provide a convincing experimental demonstration of the ease by which schemes that use GR/AGR normalization can be countered, especially by malevolent efforts (by "Eve"), as may occur in "security related scenarios" and quantum cryptography.

GR/AGR normalization and the associated gratuitously named "fair-sampling" assumption and the GR/AGR-inequality have nonetheless been used by many experiments, despite the associated proliferation of counterexamples and above noted shortcomings. (See the section "Some Experimental Results"). One possible reason for their popularity presumably is their relative ease of experimental implementation. Beyond allowing the use of strongly absorbing polarizers, no polarizers are removed under this method, and no additional normalizing data need be taken. Data collection is thereby expedited. The use of GR/AGR normalization also avoids a further difficulty associated with ternary-result apparatuses, for which polarizer removal is not readily possible. For such apparatuses, polarizer removal necessarily disturbs the collimation geometry of at least one of the channels, whereupon the polarizer-removal protocol then cannot be used.

Ursin et al. [39] in their test of a Bell's Inequality go even further than AGR by using a passive non-polarizing beam-splitter to precede a pair of ternary-result apparatuses on each side of their experiment. Each composite apparatus then has five possible results. For each ternary-result apparatus that follows the beam-splitter, Ursin et al. use a modified GR/AGR normalization scheme, where the denominator only includes coincidences associated with that ternary-result

(Footnote 2 continued)

CHSH E-inequality using Eq. (25). Subsequent experiments that use GR/AGR normalization, however, do not always meet these requirements.

apparatus. The normalization for one ternary result apparatus on one side of the experiment thus ignores coincidences occurring in the other ternary-result apparatus on the same side of the experiment. An additional facility is gained here from the use of GR/AGR normalization. Given that all four apparatus orientations needed for an evaluation of the GR/AGR R-inequality can be taken in parallel, GR/AGR normalization then allows greatly expedited data collection and a single experimental run with no required apparatus changes.

It should be noted that there is considerable confusion and misinformation in the literature on what constitutes the "CHSH inequality". Many writers mistakenly appear to believe that GR/AGR normalization is an integral part of the CHSH inequality, and fail to distinguish the CHSH E-inequality from the CHSH R-inequalities. For example, Gerhardt et al. [23] say that the CHSH E-inequality's use necessarily requires the use of the GR/AGR normalization scheme and its associated R-inequality. Giustina et al. [24] mistakenly claim that

> ...[separated apparatuses named] Alice and Bob ... each require two detectors for testing a Clauser-*Horne-Shimony-Holt inequality.*

The sections "Result Values and Expectation-Value Inequalities (E-Inequalities)"–"Garuccio and Rapisarda/Aspect Grangier Roger R-Inequalities for Real Ternary-Result Apparatuses" above all show the falsity of these claims.

Finally, it should also be noted that Christensen et al. [11], mistakenly claim that

> ...all previous experiments have had to make fair-sampling assumptions that the collected photons are typical of those emitted (this assumption is demonstrably false (Marshal et al. [31]) for many of the pioneering experiments using atomic cascades (Freedman and Clauser [20], and Aspect Dalibard and Roger [4]) and has been intentionally exploited to fake Bell violations in recent experiments (Gerhardt et al. [23]) ....

Their erroneous statement clearly does not apply to the two cascade photon experiments they quote, notably to that by Freedman and Clauser [20] and that by Aspect et al. [4], which both use the CHSH polarizer-removal protocol and CH no-enhancement assumption, that, in turn, is not demonstrably false.

Some Experimental Results

Table 28.1 lists chronologically some of the experimental tests to date of the various Bell Inequality predictions. (I apologize to the authors of references omitted from this table.) The experiment is identified in columns 1 and 2. The entangled systems and source are given in column 3, along with whether or not locality was tested. The number of apparatus channels (binary or ternary) is shown in column 4. The Bell Inequality that was tested is shown in column 5, along with the associated normalization protocol that was used. Whether an accidental background rate was subtracted is indicated in column 6, and the magnitude of the observed inequality violation is shown in column 7.

Table 28.1 Bell's inequalities—experimental tests (incomplete list). See the section "Some Experimental Results" for a discussion of these experiments

Year	Author(s); Institution(s)	System, excitation method (locality tested?)	# Channels per apparatus	Bell Inequality tested, normalization method	BkgSubtr? Violation
1972	Freedman, Clauser; U. Cal., Berkeley	Ca cascade photons, UV discharge lamp	1Det. + No Cnt. (binary)	CHSH & Freedman ≠'s, pol removal + CH no Enh.	No, 6.3σ
1973	Holt, Pipkin (unpub.); Harvard U.	Hg cascade photons, e-beam	1Det. + No Cnt. (binary)	CHSH-Freedman ≠, pol removal + CH no Enh.	Yes, None
1976	Clauser; U. Cal., Berkeley	Hg cascade photons, e-beam	1Det. + No Cnt. (binary)	CHSH-Freedman ≠, pol removal + CH no Enh.	Yes, 4.1σ
1976	Fry, Thompson; Texas A&M U.	Hg cascade photons, e-beam + laser	1 Det. + No Cnt. (binary)	CHSH-Freedman ≠, pol removal + CH no Enh.	Yes, 3.3σ
1981	Aspect, Grangier, Roger; U. Paris-Sud	Ca cascade photons, 2 lasers	1 Det. + No Cnt. (binary)	CHSH-Freedman ≠, pol removal + CH no Enh.	Yes, 13σ
1982	Aspect, Grangier, Roger; U. Paris-Sud	Ca cascade photons, 2 lasers	2 Det. + No Cnt. (ternary)	CHSH ≠ norm. by GR/AGR sum of 4 coinc. rates	Yes, 5σ
1982	Aspect, Dalibard, Roger; U. Paris-Sud	Ca cascade photons, 2 lasers (test of locality)	1 or 1 (active sw) + No Cnt. (binary)	CHSH ≠ pol removal + CH no Enhancement	Yes, 5.1σ
1987	Shih, Alley; U. Maryland	PDC (parametric down-conversion) photons	1 Det. + No Cnt. (binary)	CHSH (Freedman) ≠ pol removal + CH no Enh.	3σ
1988	Ou, Mandel; U. Rochester	PDC photons	1 Det. + No Cnt. (binary)	CHSH ≠ pol removal + CH no Enhancement	Yes, 5.8σ
1995	Kwiat et al. (6 authors); U. Innsbruck et al.	Type II PDC photons	1 Det. + No Cnt. (binary)	CHSH ≠ norm. by GR/AGR sum of 4 coinc. rates	No, 102σ
1998	Weihs et al. (5 authors); U. Innsbruck	Type II PDC photons (test of locality > 400 m)	2 Det. + No Cnt. (ternary-active sw)	CHSH ≠ norm. by GR/AGR sum of 4 coinc. rates	No, 37σ
1998	Tittel et al. (4 authors); U. Geneva	PDC photons, Franson Energy-time entanglement (test of locality > 10 km)	2 Det. + No Cnt. (ternary via beam-splitter)	CHSH ≠ norm. by GR/AGR sum of 4 coinc. rates	No, 16σ Yes, 25σ

(continued)

Table 28.1 (continued)

Year	Author(s); Institution(s)	System, excitation method (locality tested?)	# Channels per apparatus	Bell Inequality tested, normalization method	BkgSubtr? Violation
2001	Rowe et al. (7 authors); NIST	Induced entanglement of 2 Be ions (spatially unresolved)	2 Det. (binary via active switch)	CHSH $\neq$, norm. by # heralded events	No, 8.3σ
2006	Matsukevich et al. (6 authors); GeorgiaTech	Induced entanglement of 2 Rb clouds	2 Det. + No Cnt. (ternary)	CHSH $\neq$, norm. by GR/AGR sum of 4 coinc. rates	No, 5.3σ
2007	Ursin et al. (18 authors); U. Vienna et al.	PDC photons (test of locality through 144 km of air)	4 Det. + No Cnt. (fivefold via passive switch/beam-splitter)	CHSH $\neq$ norm. by GR/AGR selected sum of 4 (out of 16) coinc. rates	No, 14σ
2008	Matsukevich et al. (5 authors); U. Maryland	Induced entanglement of trapped Yb ion & photon of 2 trapped Yb ions	2 actively selected Det. (binary)	CHSH $\neq$ norm. by # heralded events	No, 27σ No, 3.1σ
2009	Ansmann et al. (13 auth.); U. Cal., Santa Barbara	Induced entanglement of Josepson phase qubits	1 Det. (binary via active switch)	CHSH $\neq$ norm. by # heralded events	No, 244σ
2012	Hofmann et al. (7 authors); Ludwig-Maximilians U. et al.	Induced entanglement of 2 trapped Rb atoms	2 actively selected Det. (binary)	CHSH $\neq$ norm. by # heralded events	No, 2.1σ
2013	Giustina et al. (12 authors); U. Vienna et al.	PDC photons (non-maximally entangled)	1 Det. + No Cnt. (binary)	CH $\neq$ norm. by # heralded events & singles rates	No, 69σ
2013	Christensen et al. (14 authors); U. Illinois Urbana-Champaign et al.	PDC photons (non- maximally entangled)	1 Det. + No Cnt. (binary)	CH $\neq$ norm. by # heralded events & singles rates	No, 7.5σ

All of these experiments except one—that by Holt and Pipkin [27], (see also Holt [26])—agree with the associated predictions by quantum mechanics. Clauser [14] repeated the Holt and Pipkin experiment with only a few minor changes and obtained the opposite results, i.e. results in agreement with quantum mechanics. The earliest experimental tests by Freedman and Clauser [20], Clauser [14], Fry and Thompson [21], Aspect, Grangier, and Roger [2], and Aspect, Dalibard, and Roger [4] all used photons emitted by an atomic cascade, and also used the CHSH polarizer-removal protocol along with the CH no-enhancement assumption. GR/AGR normalization via Eq. (25) of the CHSH E-inequality (14) was used in the experiments by Aspect, Grangier, and Roger [3], Kwiat et al. [29], Weihs et al. [40], Tittel et al. [38], Ursin et al. [39], and Matsukevich et al. [32].

The experiments by Aspect, Dalibard, and Roger [4], Weihs et al. [40], Tittel et al. [38] and Ursin et al. [39] all changed the analyzers while the entangled-state photons were in flight, thereby providing a direct realization of Bohm and Aharonov's [10] locality-test. The experiment by Tittel et al. [38] is noteworthy in that the photons use energy-time entanglement, rather than polarization-state entanglement.

The Fry and Thompson [21] experiment was the first to use tunable laser excitation of the source atomic cascades, thereby providing a dramatic boost in count rates over previous experiments. The use of parametric down conversion in a crystal as a source of entangled-state photons was first offered in 1988 by Shih and Alley [36], and by Ou and Mandel [34]. It provides a further dramatic boost to count rates when compared to those emitted by atomic cascade decays. Kwiat et al. [29] further enhanced count rates via the use of Type II parametric down conversion.

The experiment by Rowe et al. [31] was the first to violate the "heralded" CHSH inequality using a heralded source, and thereby to avoid the normalization loophole. However, in their experiment, light from the two entangled-state Beryllium ions is commingled indistinguishably in a single detector. By contrast, the basic locality postulates associated with Bell's Theorem prototype configurations call instead for a pair of widely separated independent detectors with no worry about their possible intercommunication. Unfortunately, the two ions in the Rowe et al. experiment were seemingly in intimate communication with each other, and even share the same probe laser light that was used to determine their excitation states. Correspondingly, it is not clear if there were any interfering interference effects (classical, quantum mechanical, or otherwise) from unresolved light emitted by both, ions. Interference effects were indeed observed earlier in a similar experiment by Eichmann et al. [18]. Rowe et al. do note, however that the ions' separation was wide enough that associated Young's fringes average out. However, this fact does not rule out some other perhaps non-quantum-mechanical and/or non-classical interfering interference effect. Recall that Bell's Inequality tests seek to determine whether or not quantum-mechanics is correct, and/or even whether or not any of the physics generally assumed to govern the formation of interference fringes is correct. Thus, given the level of generality required for such tests, such claims of independence are not fully reassuring, and such assumed physics cannot be relied upon here.

The experiment by Ansmann et al. [1] entangled a pair of Josephson phase qubits to violate the heralded CHSH inequality. Since the entangled qubits were only 3.1 mm separated, intimately coupled, and indistinguishably probed, this experiment is subject to similar criticisms to those regarding Rowe et al.

More convincing violations of the CHSH inequality using a heralded source with well separated apparatuses were subsequently reported by Matsukevich et al. in [33] in 2008, and by Hofmann et al. [25] in 2012. The experiment by Matsukevich et al. [33] entangled a pair of Yb^+ remotely trapped ions. The experiment by Hofmann et al. [25] entangled a pair of widely-separated remotely trapped Rubidium atoms.

Finally, the experiment by Giustina et al. [24] was the first to directly violate the CH inequality. It was followed very shortly by a similar experiment by Christensen et al. [11]. Closure of both the normalization loophole and the locality loophole simultaneously in a single experiment has not yet been done.

Appendix: Local Realism

Local Realism was first explicitly defined by Clauser and Horne (CH) [13] in 1974, and further clarified in a series of papers by Bell et al. [8] in 1976–1977 and by Clauser and Shimony [15] in 1978. CH originally called the theories governed by it, "Objective Local Theories". Clauser and Shimony renamed these theories "Local Realism". Local Realism is the combination of the philosophy of realism with the principle of locality. The locality principle is based on special relativity. It asserts that nature does not allow the propagation of information faster than light to thereby influence the results of experiments. Without locality, one must contend with paradoxical causal loops, as are now popular in science fiction thrillers involving time travel. Upholding locality is effectively a denial of the reality of causal loops. Equivalently, it is the assertion that history is single valued. Realism is a philosophical view, according to which external reality is assumed to exist and have definite properties, whether or not they are observed by someone. Bell's Theorem, and the experimental predictions made by the associated CHSH and CH Bell's Inequalities, along with the associated experimental tests of these predictions, show that any theory that combines Realism and locality, must be in observed disagreement with these experiments. Consequently, it can now be asserted with reasonable confidence that either the thesis of Realism or that of locality (or perhaps even both) must be abandoned.

Another way of describing what we mean by Realism here is to say that it specifies that nature consists of "objects", i.e. stuff with "objective reality". Realism assumes that objects exist and have inherent properties on their own. It does not require that these properties fully determine the results of an experiment locally performed on said object. Instead, in a possibly non-deterministic world, it simply allows the properties of an object to influence the probabilities of experiments being performed on it. There is also nothing in our specification that prohibits an act of observation or measurement of an object from influencing. perturbing and/or even destroying said properties of the object.

Realism thus assumes that an object's properties determine minimally the probabilities of the results of experiments locally performed on it. Realism, under the additional constraint of locality, i.e. Local Realism, then assumes that the results of said experiments do not depend on other actions performed far away by someone else, especially when those actions are performed outside of the light-cone of the local experiment.

Properties, as referred to here, are what John Bell called Beables, and what Einstein et al. [19] called "elements of reality". The properties of an object constitute a description of the stuff that is "really there" in nature, independently of our observation of it. When we perform a "measurement" of these properties, we don't really need to know what we are actually doing, or what we are really measuring. What we are assuming is that what is "really there" somehow influences what we observe, even if said influence is inherently stochastic and/or perhaps irreproducible from one measurement to the next.

Recall that Einstein et al. [19] attempted to define an object's properties as something that one can measure, but they further required that the measurement result be predictable with certainty. However, given Ben Franklin's observation that the only predictions that are certain in life are for death and taxes (see the section "Bell's 1964 E-Inequality for Idealized Binary Result Apparatuses"), said definition becomes meaningless, because it describes nothing that can ever occur in reality, (unless, of course, said properties are equivalent to death and taxes). Our definition is very much looser and requires no predictions with certainty.

Precisely how does one define an object with such extreme generality? For the purposes of Local Realism and its tests via Bell's Theorem, a purely operational definition of an object suffices. An object (or collection of objects) is stuff with properties that one can put inside a box, wherein one can then perform measurements inside said box and get results whose values are presumably influenced by the object's properties. What then is a box? A box is defined as a closed three-dimensional Gaussian surface,[3] inside of which one can perform said measurements of said properties. For Local Realism, such a box becomes a four-dimensional Gaussian surface consisting of the backward light cone (extending to $t = -\infty$) enveloping a three dimensional box, that contains the object(s) being measured, at the time that they are being measured.

Familiar examples of "classical" objects that can be put into boxes are galaxies, stars, airplanes, shoes, trapped clouds of atoms, single trapped atoms, electrons, y-polarized photons, a single bit of information, etc. All of these can be put into a box and have their properties (e.g. color, mass, charge, etc.) measured. Or can they? Via Bell's Theorem experiments, one may ask—are there examples of objects that

[3]Gauss showed that a "Gaussian surface" is one that divides all of space into two disjoint volumes, wherein one of these volumes may be called the inside, and the other the outside.

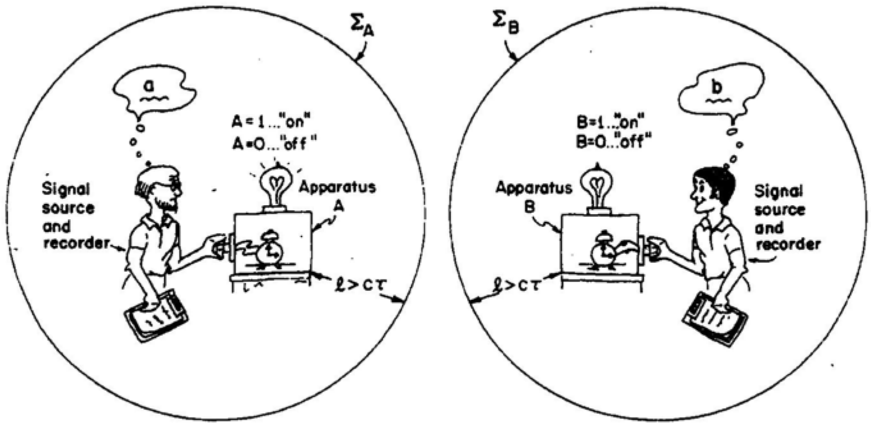

Fig. 28.4 Worst-case set of required elements for a Bell's Inequality experiment. This figure was first presented by the author at the 1976 International "Ettore Majoranna" Conference in Erice, Sicily on "Experimental Quantum Mechanics". The conference was organized by John Bell, Bernard d'Espagnat, and Antonino Zichichi. With present-day jargon, the characters labeled "Signal source and recorder" might now be named Alice and Bob.

cannot be put inside such boxes?[4] If so, such objects cannot be described by Local Realism. Furthermore, if there are parts of nature that cannot be described by Local Realism, then Local Realism must be discarded as a description of all of nature. Sadly, (for Local Realism advocates[5]) the individual particles comprising a quantum-mechanically entangled pair of particles are parts of nature that cannot be described by Local Realism.

Figure 28.4 shows the worst-case set of required elements for a fully loophole-free Bell's Inequality experimental test. Two objects and associated binary-result apparatuses are each contained in associated boxes that are space-like separated at the time of the measurement events. The apparatuses measure quantum-mechanically entangled pairs of particles. The boxes are labeled Σ_A. and Σ_B in the Figure. Each box contains a signal recorder and signal source. Each signal source generates via the free-will of an observer an appropriate apparatus parameter setting. The two settings are respectively called **a** and **b**. Each box contains a clock that permits synchronized measurements in the two boxes of the object pairs, that were emitted in the past and that have propagated into these boxes at subliminal speed for measurement by the apparatuses.

[4]The fact that the simplest possible object—a single bit of information—cannot be put into a "box", in turn gives rise to the field of quantum information. It also calls into question a claim often made by general relativists that information is always contained within a given spatial volume and cannot be destroyed.

[5]John Bell and I have both confessed to being former advocates of Local Realism.

An important issue discussed in "An Exchange on Local Beables", (Bell, Shimony, Horne, and Clauser [8]), is that the apparatus parameters **a** and **b** must be generated independently, for example by the presumed free will of the observers, and are not to be counted as part of the objective reality being measured.

To derive a Bell Inequality, one then needs to assume the following requirements for a Local-Realistic theory:

(1) The probability of obtaining the measured result A in box Σ_A may depend on all of the stuff (objects) that are inside the box at the time of the measurement, including any stuff that may have propagated into the box at a velocity less than or equal to the speed of light since the beginning of time.
(2) The probability of obtaining the measured result A in box Σ_A may depend on the freely chosen apparatus parameter **a**.
(3) Locality, however, prohibits the probability of obtaining the measured result A in box Σ_A from depending on the apparatus parameter **b** that was freely chosen in the space-like separated box Σ_B.
(4) Locality, similarly, prohibits the probability of obtaining the measured result A in box Σ_A from depending on the result B, as measured in box Σ_B, which, of course, is allowed to depend on the parameter **b**.
(5) Similar reciprocal permissions and prohibitions like (1)–(4) govern the probability of obtaining the measured result B in box Σ_B.

Surprisingly, that's all you need to derive the CH (and CHSH) inequality and thereby to constrain and test Local Realism!

References

1. M. Ansmann et al., Science **461**, 504–506 (2009)
2. A. Aspect, P. Grangier, G. Roger, Phys. Rev. Lett. **47**, 460 (1981)
3. A. Aspect, P. Grangier, G. Roger, Phys. Rev. Lett. **49**, 91 (1982)
4. A. Aspect, J. Dalibard, G. Roger, Phys. Rev. Lett. **49**, 1804–1807 (1982)
5. F. Belinfante, *A Survey of Hidden Variables Theories* (Pergamon, Oxford, 1973)
6. J. Bell, Physics **1**, 195 (1964)
7. J. Bell in *Foundations of Quantum Mechanics, Proceedings of the International School of Physics "Enrico Fermi"*, ed. by B. d'Espagnat (Academic Press, New York, 1971)
8. J. Bell, A. Shimony, M. Horne, J. Clauser (1976–1977) Epistemological Letters (Association Ferdinand Gonseth, Institut de la Methode, Case Postale 1081, CH-2501, Bienne.) republished as *An Exchange on Local Beables* in Dialectica, **39**, 85–110 (1985)
9. D. Bohm, *Quantum Theory* (Prentis Hall, Englewood Cliffs, NJ, 1951)
10. D. Bohm, Y. Aharonov, Phys. Rev. **108**, 1070 (1957)
11. B. Christensen et al., Phys. Rev. Lett. **111**, 130406–130409 (2013)
12. J. Clauser, M. Horne, A. Shimony, R. Holt, Phys. Rev. Lett. **23**, 880–884 (1969)
13. J. Clauser, M. Horne, Phys. Rev. D **10**, 526–535 (1974)
14. J. Clauser, Phys. Rev. Lett. **36**, 1223 (1976)
15. J. Clauser, A. Shimony, Rep. Prog. Phys. **41**, 1881–1927 (1978)

16. J. Clauser in *Quantum [Un]speakables, From Bell to Quantum Information, Proceedings of the 1st Quantum [Un]speakables Conference*, ed. by R. Bertlmann, A. Zeilinger (Springer, Berlin, 2002), pp 61–96
17. P. Eberhard, Phys. Rev. A **47**, R747–R750 (1993)
18. U. Eichmann et al., Phys. Rev. Lett. **70**, 2359–2362 (1993)
19. A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. **47**, 777–780 (1935)
20. S. Freedman, J. Clauser, Phys. Rev. Lett. **28**, 938–941 (1972)
21. E. Fry, R. Thompson, Phys. Rev. Lett. **37**, 465 (1976)
22. A. Garuccio, V. Rapisarda. 1981. Nuovo Cimento **A65**, 269 (1981)
23. L Gerhardt et al., Phys. Rev. Lett. **107**, 170404 (2011)
24. M. Giustina et al., Nature **497**, 227–230 (2013)
25. J. Hofmann et al., Science **337**, 72–75 (2012)
26. R. Holt, Ph. D. thesis, Harvard Univ. (1973)
27. R. Holt, F. Pipkin, unpublished preprint, Harvard University. (1973)
28. C. Kocher, E. Commins, Phys. Rev. Lett. **18**, 575 (1967)
29. P. Kwiat et al., Phys. Rev. Lett. **75**, 4337–4341 (1995)
30. W. Lamb, M. Scully, in *Polarization: Matiere at Rayonnement*, Société Française *de Physique*, Presses Universitaires de France, Paris (1968)
31. T. Marshall, E. Santos, F. Selleri, Phys. Lett. A **98**, 5–9 (1983)
32. D. Matsukevich et al., Phys. Rev. Lett. **96**, 030405 (2006)
33. D. Matsukevich et al., Phys. Rev. Lett. **100**, 150404 (2008)
34. Z. Ou, L. Mandel, Phys. Rev. Lett. **61**, 50–53 (1988)
35. M. Rowe et al., Nature **409**, 791–794 (2001)
36. Y. Shih, C. Alley, Phys. Rev. Lett. **61**, 2921–2924 (1988)
37. A. Shimony in *Foundations of Quantum Mechanics, Proceedings of the International School of Physics "Enrico Fermi"*, ed. by B. d'Espagnat (Academic Press, New York, 1971)
38. W. Tittel et al., Phys. Rev. Lett. **81**, 3563–3566 (1998)
39. R. Ursin et al., Nat. Phys. **3**, 481–486 (2007)
40. G. Weihs et al., Phys. Rev. Lett. **81**, 5039–5043 (1998)
41. E. Wigner, Am. J. Phys. **38**, 1005–1009 (1970)